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A muylti-year ammunition procurement model for non-nuclear ordnance

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THESIS

A MULTI-YEAR AMMUNITION PROCUREMENT MODEL FOR NON-NUCLEAR ORDNANCE

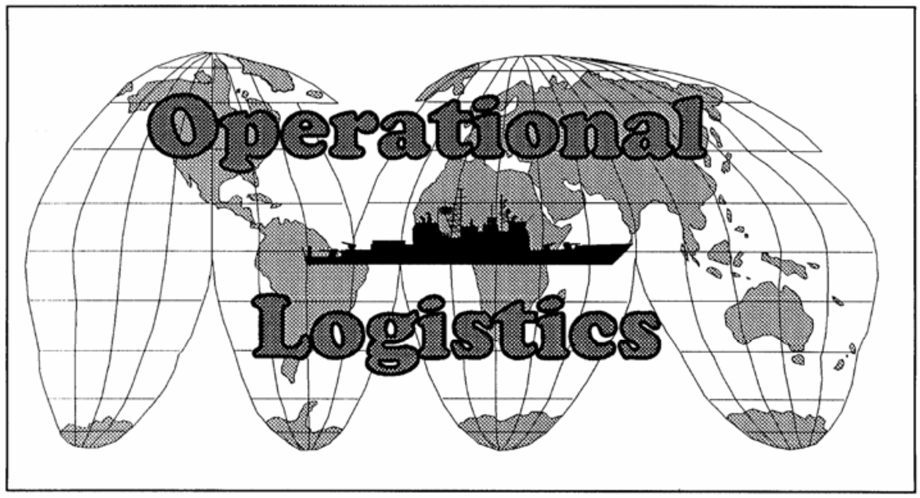
by

Charles E. Hurst Jr.

September 2004

Thesis Advisor: W. Matthew Carlyle
Second Reader: Gerald G. Brown

Amateurs discuss strategy,
Professionals study logistics



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This report reformulates AIM and dramatically improves response times for almost all instances. We report analyses using AIM, involving a variety of budgeting and inventory scenarios.

AIM is now a fast, flexible tool that can handle a wide range of budget and requirements scenarios in a manner that was previously impossible. Decision makers can now develop a procurement plan that effectively and efficiently meets the ordnance needs of the world's most powerful Navy.

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A MULTI-YEAR AMMUNITION PROCUREMENT MODEL FOR NON-NUCLEAR ORDNANCE

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ABSTRACT

The Assessment and Investment Model (AIM) introduced in 2003 a fiscally constrained ordnance procurement model to plan procurements of the most capable inventory of munitions while attempting to meet annual Navy Non-Nuclear Ordnance Requirements (NNOR). AIM is the first analytical planning tool to incorporate fiscal constraints, to use true optimization to guide procurement policy, and to establish a quantifiable measure of overall inventory capability.

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EXECUTIVE SUMMARY

Every year the Navy is presented with budgets and projected budgets for several out-years with which they must develop a long-term munitions procurement plan guided by the Navy Non-Nuclear Ordnance Requirements (NNOR).

Developing an effective and efficient munitions procurement plan is very important for the Navy due to the significant investment involved and the operational importance of the ordnance. A budget plan must be developed that maximizes the capabilities of the many types of munitions we purchase so that our war fighter is provided with the most capable and effective munitions inventory possible.

U.S. Navy Non-nuclear Ordnance Requirements (NNOR) are established by a complex, expensive process involving many stakeholders in a mostly manual planning exercise.

Decision makers currently use NNOR requirements to determine subjectively what munitions to purchase in order to provide the greatest capability for the overall inventory. This is sensitive to factors ranging from munitions pricing to the personalities and preferences of the decision makers. Such a process cannot guarantee a munitions procurement plan that provides the greatest inventory capability for the dollars invested.

Having recognized these opportunities for improving NNOR planning, Major John Bruggeman developed the Assessment and Investment Model (AIM) in 2003. AIM is a

simple and straightforward tool with which to develop reliable and objective procurement decisions.

Bruggeman uses tier levels to break each weapons class into discrete levels of capability; this provides meaningful summaries of inventories, and allows for insightful analyses using well-known and accepted methods such as “stoplight diagrams.”

AIM is a versatile and powerful planning tool. However, AIM requires a relatively long computational cycle. To develop an estimate that can be certified to be within 20% of the best possible optimal solution requires a runtime of more than an hour on a fast desktop computer. While this is much faster than the current manual procurement process (which can take days or even months and never yield a provably optimal solution), it does not lend itself to extensive experimentation.

We present a reformulation that executes more quickly and provides more precise output. This supports multiple runs to explore alternate munitions planning options. It also encourages decision makers to explore the impact of changing munitions and mission priorities.

We have discovered that the long AIM solution times can be explained by the original tier level definitions of AIM. Tier levels have been restricted to be expressed in integer values. We reformulate tier levels as a piecewise linear function of munition inventory. This makes AIM significantly faster, and it provides better guarantees of optimality than its predecessor.

Our reformulation of the model is an unquestioned success in terms of speed. However, we must also ensure the recommendations from the model remain valid and comparable to that of its predecessor in terms of overall improvement in munitions capability. To do this we compare the capabilities of nineteen test munitions at the end of an eight-year planning period as generated using the original AIM model and our reformulation. The results show different munitions are recommended in various years of the plan, but overall the end results of the two models are comparable. The overall measurement of munitions capability, the minimum tier level across all munitions, is consistently higher in the reformulated model.

We illustrate the usefulness of our new reformulation with four hypothetical examples. In the first scenario we examine a case where a particular weapon has a beginning inventory suddenly dropped to zero. When compared to the baseline procurement plan (level procurement funding throughout the planning horizon and a given level of maintenance funding), we observe an increased number of procurements earlier in the eight-year plan as one would logically expect in order to attempt to replenish stock to satisfy NNOR.

Next, we consider a scenario where some portion of the procurement budget is moved into the budgeting out-years. While the total budget over the eight-year period remains constant, funding has been shifted from each of years one through four to years five through eight. Here we see that, compared to the baseline, munitions capability, as measured by tier level achieved, lags behind the baseline, level funding plan.

The third scenario considers a case where procurement funding is higher early in the planning years, essentially the opposite of our second scenario. Here funds are moved from the last three years of the budget to the first three. The resulting plan is surprising in that it shows no noticeable improvement in capability during the early years despite increased funding. While not what we expected, such a result is useful in that it might trigger the decision maker to look further into the scenario to determine if other factors might be affecting the plan. In this case, the model saves the earlier money to make a larger purchase in middle years, when it can have more impact on overall capability.

The final scenario we examine contains a year one spike in maintenance funding. We observe no dramatic effect, overall, other than a more gradual capability increase over the baseline plan. A logical next step with this scenario might be to conduct a sensitivity analysis on the level of funding increase to determine how large such an increase must be to have a significant affect on capability. Such an analysis can now be easily done given the improved speed of our model.

We have developed an improved version of AIM that, combined with a user-friendly spreadsheet interface, provides the decision maker an easy to use, extremely fast and accurate tool with which to explore multiple budgetary and requirements scenarios with minimal time and effort. The end result provides the decision maker with a quantitative approach for addressing the ordnance requirements of the U.S. Navy.

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I. INTRODUCTION

A. BACKGROUND

Munitions procurement is a long-term planning problem and, as such, the Navy is presented with budgets and projected budgets for several years with which to develop a long-term procurement plan around the Navy Non-Nuclear Ordnance Requirements (NNOR). NNOR provides official Department of the Navy (DoN) estimates of munitions requirements which are then to be used in developing procurement budgets [OPNAVINST 8011.9A, 1989]. Frequently, these budgets are significantly lower than what would be required to purchase all weapons recommended.

Munitions procurement is a very important problem for DoN as well as the entire Department of Defense (DoD) due to the dollars involved and the military importance of the commodity with which we are dealing. The DoN weapons procurement budget this year is in excess of $2B [DoN Report, 2005] and covers approximately 48 weapons classes. An effective budget plan must be developed that maximizes the capabilities of the multiple munitions we purchase so that the war fighter at the tip of the spear is provided with the most capable and effective munitions inventory possible within unavoidable fiscal limitations.

1. Current Method

U.S. Navy non-nuclear ordnance requirements are established by a complex, expensive and labor intensive process. NNOR involves multiple stakeholders in a process that is primarily manual. NNOR ignores budget restrictions. This leads to recommendations that are, most likely, completely unachievable when budgets are eventually set.

Once NNOR requirements are established, the procurement process as it exists now is driven by highly-subjective allocation priorities. For example, the first priority is to replace unexpected expenditures from the previous year. The next consideration is given to expected expenditures for the current year based on projections from NNOR. At this stage providing munitions for training is generally the first consideration. After that, minimum production quantities are considered in order to maintain the industrial base. Should there be funds remaining at that point, munitions that are farthest from their desired inventory levels are targeted for procurement [Fahringer, 2003].

In the present system, decision makers take the NNOR requirements and determine subjectively what munitions to purchase in order to provide the greatest capability for the overall inventory. Obviously, this process is extremely sensitive to factors ranging from munitions pricing to the personalities and preferences of the decision makers involved. Such a process will not necessarily provide the Navy with the most capable munitions procurement plan, especially in the absence of an agreed-upon measure of capability. There is no single, monolithic justification for the plan as a whole, it simply emerges as a set of allocations with scant justification for why each suggested procurement appears.

2. Assessment and Investment Model

The Assessment and Investment Model (AIM) [Bruggeman, 2003] seeks to improve munitions planning. AIM provides the decision maker with a simple and relatively straightforward tool with which to develop reliable and objective procurement decisions. The procurements suggested by AIM are optimal or near-optimal for a measure of effectiveness that is based on mission capability and priorities agreed upon by decision makers. In contrast to the current method of procurement planning, AIM is a prescriptive model that utilizes a detailed mathematical optimization model (see Appendix) to arrive at an optimal munitions procurement plan.

A central concept of the AIM formulation is that of procurement tiers; a clear and objective measure of inventory capability. Bruggeman uses tier levels to break the weapons classes into levels of capabilities that allow for insightful analysis using well-known, accepted methods such as stoplight diagrams.

B. PROBLEM DEFINITION

AIM requires a relatively long computational cycle. To develop an estimate that is provably within 20% of the optimal solution requires a runtime of more than an hour on a fairly powerful desktop computer. While this much faster than the current manual system (which can take days or even months and never yields a provably optimal solution), it does not lend itself to experimentation.

We seek a reformulation that executes more quickly and provides more precise output. This would support multiple runs to explore several alternate munitions planning options. It would also encourage decision makers to assess the impact of changes to mission priorities, budgets, etc.

We concentrate on reformulating the tier level constraints and variables and, ultimately their implementation in the General Algebraic Modeling System (GAMS) [Brooke Kendrick, and Meeraus, 1998]. Bruggeman expresses the tier level of a munition as a whole number. We suspect this artificial restriction is unnecessary and needlessly complicates the calculations of the model without adding insight into the problem. We relax this integrality restriction, thus allowing munitions capability to fall in a continuous band between the lower and upper limits of the tiers as determined by the end user. This reformulation simplifies the model and dramatically reduces solve times.

II. MODEL MODIFICATION

A. TIER LEVEL REFORMULATION

1. Tier Level Description

A central feature of AIM is a tier level that provides both the model and decision maker with a clear and objective measure of inventory capability. As shown in Table 1, the NNOR Total Munition Requirement (TMR) is composed of four mission areas; Training and Testing, Current Operations/Forward Presence, Combat and Strategic Readiness. Each weapon class is assigned a primary, secondary, and a tertiary mission, and, based on the total number of that weapon in inventory, a letter grade is assigned, see Table 2, based on how much of each mission is covered for that weapon. Assigned grades represent the increasing capability of specific munitions as their inventory increase as illustrated in Figure 1.

TMR =	TTR +	CO/FPR +	CR +	SRR

Table 1. NNOR components
The NNOR Total Munition Requirement (TMR) consists of the Training and Testing Requirement (TTR), Current Ops/Force Protection Requirement (CO/FPR), Combat Requirement (CR), and Strategic Reserve Requirement (SRR).

Mission Capability Score	Inventory (as a % of Mission Requirement)
Level F (None)	0%
Level E (Basic)	40%
Level D(Intermediate)	50%
Level C (Advanced)	60%
Level B (Superior)	70%
Level A (Full)	100%

Table 2. Mission Capability Scores

The capability provided by a munition inventory is represented by a series of discrete jumps in relative inventory count. (From [Bruggeman, 2003])

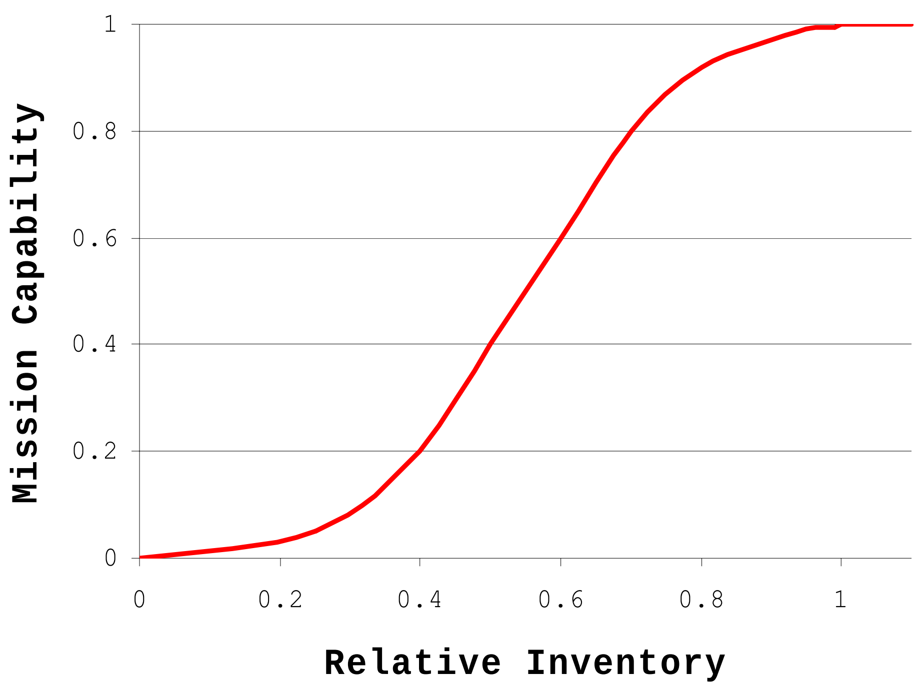

Figure 1. Mission Capability Score as a function of Inventory Count

A typical relationship between the mission-related capability score of a munition and the inventory count as a proportion of TMR illustrates reduced marginal utility at the extreme inventory levels. (From [Bruggeman, 2003])

The letter grade in each of the three missions led Bruggeman to derive the tier levels shown in Table 3. Essentially, each of the 16 tier levels proposed by Bruggeman corresponds to a specific total number of weapons of that class.

Tier Level	**Mission Areas**			**Treaty Requirement**
	Primary	**Secondary**	**Tertiary**	
1	F	F	F	A
2	E	F	F	A
3	D	F	F	A
4	D	E	F	A
5	D	E	E	A
6	C	E	E	A
7	C	D	E	A
8	C	D	D	A
9	B	D	D	A
10	B	C	D	A
11	B	C	C	A
12	A	C	C	A
13	A	B	C	A
14	A	B	B	A
15	A	A	B	A
16	A	A	A	A

Table 3. Tier level formulation. The capability of a particular munition progresses through the tier levels as inventory satisfies a greater portion of each of its assigned mission areas. (From [Bruggeman, 2003])

The correspondence between tier levels and inventory numbers depends on the specific weapon and its individual mission requirements by mission area (as determined by NNOR). As the inventory level of a particular munition increases to the point of satisfying a greater part of its mission requirements, the weapon is assigned a higher letter grade ranging from a low of “F” to a high of “A”. The combination of qualifications in each mission area provides for an overall evaluation of an individual munition's capability that is expressed in a tier level designation.

Tier levels appear in the AIM objective function, as AIM seeks to maximize the minimum tier level achieved across the spectrum of munitions.

2. Cumulative Design

The tier level constraints of the Bruggeman model found in (equations 24 through 28 of the Appendix) rely on a binary variable, $CUM\_TIER\_REACHED_{m,ty}$, that records when the inventory level of a munition reaches or exceeds a certain tier level.

This approach, while simple and very useful, restricts tier levels to integer values only. Though the capability of a munition might actually lie between one tier breakpoint and another, it would only be recorded at the lower of the two.

Because tier levels are an artificial construct, the requirement that we only recognize achievement of integer levels seems overly restrictive.

3. Piecewise Linear Design

Bruggeman[2003] recognizes this deficiency and suggests relaxing the integrality restriction. He conjectures that this will dramatically improve solution times.

Given both time and opportunity, we have pursued this relaxation. This requires a reformulation of the four tier level constraints of Bruggeman and the addition of a fifth constraint to accurately model the tier structure. The new formulation combines each tier level segment into a piecewise linear function. This is achieved by introducing variables and equations for each munition that allow the exact fractional tier level to be calculated by creating a convex combination of the two proximate tier level breakpoints the current inventory lies between.

Figure 2 illustrates integer tier levels and our continuous relaxation of these.

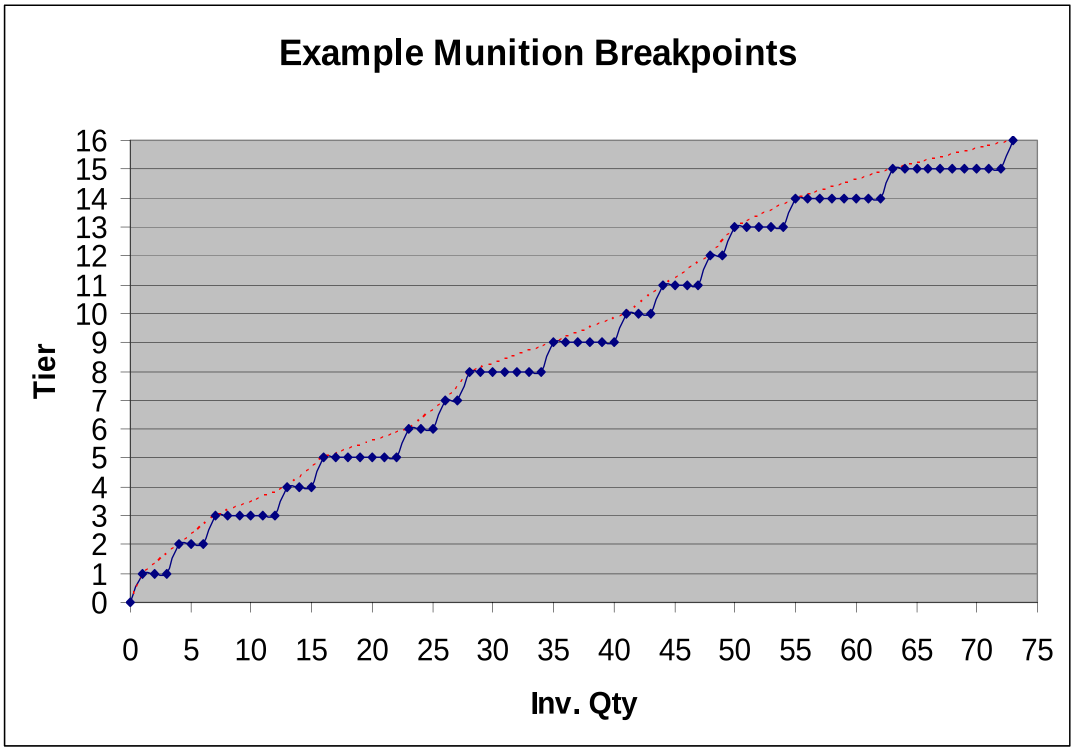

Figure 2. Tier breakpoints: Integer tier levels

The dotted stepwise function shows integer tier levels, while the piecewise linear function continuously interpolating between tier level breakpoints shows our relaxation.

The discretized tier levels hide any benefit of small increases in inventory toward the next breakpoint; AIM only credits discrete jumps into a higher tier.

We replace the binary variables that cumulatively calculate tier levels with a new set of non-negative continuous variables to represent partial fulfillment of tiers. These new variables, λ_i, are multipliers for creating a convex combination of adjacent tier level breakpoints and, thus, identify exactly where inventory capability lies within a tier. Figure 3 illustrates our concept.

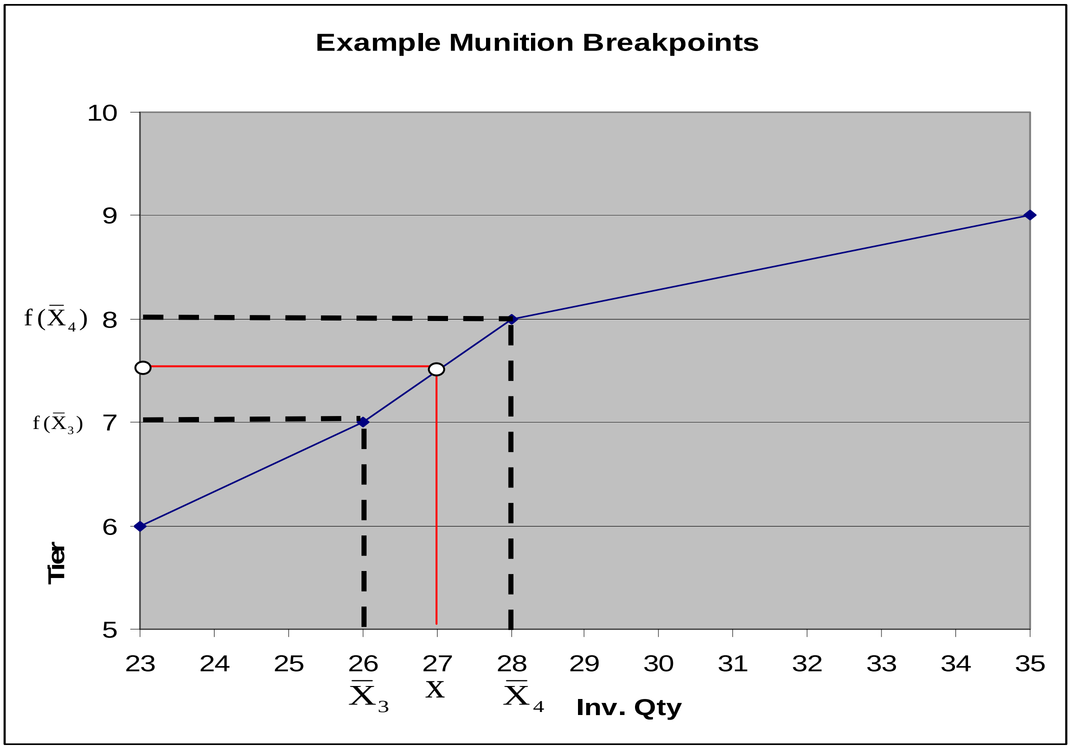

Figure 3. Piecewise linear concept
An inventory of 27 weapons lies midway within a tier bounded by 26 and 28 weapons. Rather than credit this with 7, (the score of the lower complete tier breakpoint), we interpolate and use the intermediate score of 7.5.

For tier i we have $f(\lambda_i x_i + \lambda_{i+1} x_{i+1}) \equiv \lambda_i f(x_i) + \lambda_{i+1} f(x_{i+1})$ where λ_i + λ_{i+1} = 1 and setting λ_i and λ_{i+1} appropriately (i.e. based on x) yields the correct value for $f(x)$.

In Figure 3 the current number of munitions, x, is 27. The closest breakpoints are $\bar{X}_3 = 26$ and $\bar{X}_4 = 28$; x lies halfway between these breakpoints so we have $x = \frac{1}{2}\overline{X}_3 + \frac{1}{2}\overline{X}_4$, or $\lambda_3 = \frac{1}{2}$, $\lambda_4 = \frac{1}{2}$, and the resulting capability is halfway between the two respective tier levels: $\frac{1}{2}f(\overline{X}_3) + \frac{1}{2}f(\overline{X}_4) = \frac{1}{2}(7+8) = 7.5$. Figure 4 illustrates how Figure 2 changes using the piecewise linear interpolation vice integer tier levels.

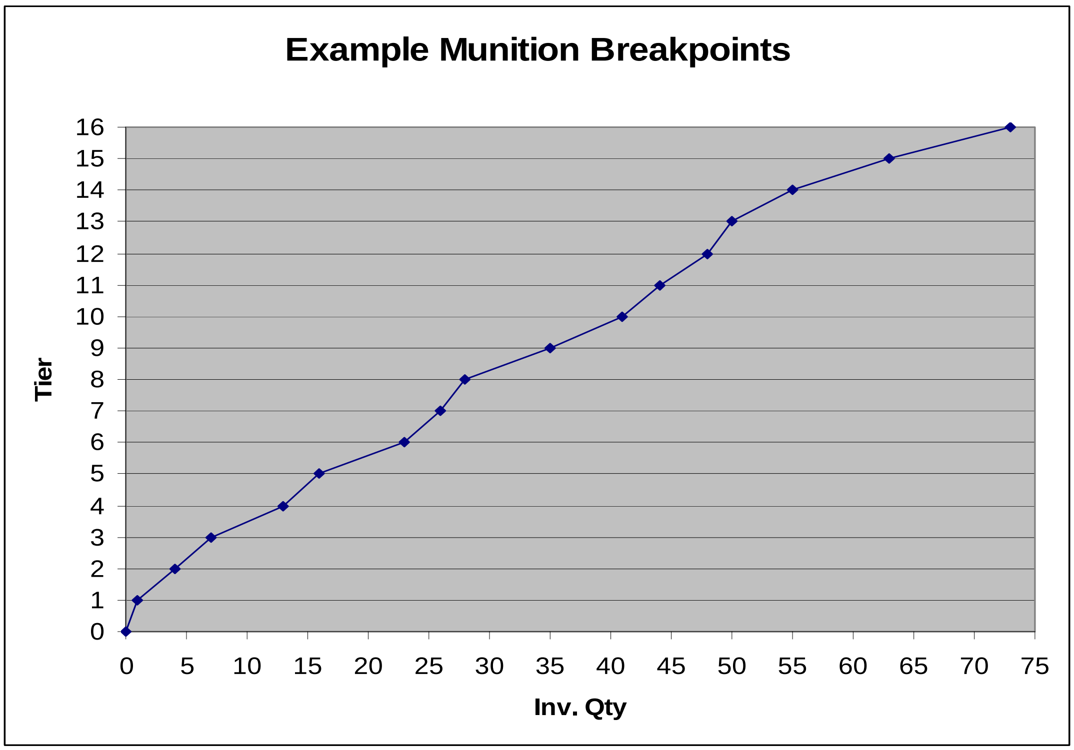

Figure 4. Tier breakpoints: Continuous interpolated tier levels

Given some inventory quantity, each proximate pair of breakpoints is used to linearly interpolate a tier score.

B. SUMMARY OF REFORMULATED CONSTRAINTS AND VARIABLES

1. Deleted Variables

We no longer need to use the $CUM\_TIER\_REACHED_{m,ty}$ variable of Bruggeman.

2. New Variables

Our reformulation of AIM requires the addition of three new variables.

$TIER\_ACHIEVED_{m,y}$	Tier level achieved by munition *m* in year *y*. In our formulation, this is now a continuous, non-negative variable that identifies the capability of a munition inventory.
$LAMBDA_{m,t,y}$	A continuous, non-negative variable representing the weight placed on tier breakpoint *t* and used to calculate the exact capability position of a munition whose inventory level lies between two integer breakpoint tier levels *t* and *t+1*.
$TIER\_INDICATOR_{m,t,y}$	Binary variable, 1 if munition *m* is in tier *t* in year *y*. This variable locates the munition capability within a tier level range. Combined with the breakpoint weights, the exact capability of the munition inventory can then be identified.

3. Deleted Constraints

The constraints given below and identified in the Appendix as (24)-(27) are now obsolete in our reformulation.

$$\mathrm{ACTIVE\_INV}_{m,y} \geq \mathrm{tier\_lvl}_{m,'1',y} * \mathrm{CUM\_TIER\_REACHED}_{m,'1',y} + \sum_{t=2}^{num\_tiers} \left[\mathrm{tier\_lvl}_{m,t,y} * \begin{pmatrix} \mathrm{CUM\_TIER\_REACHED}_{m,t,y} \\ -\mathrm{CUM\_TIER\_REACHED}_{m,t-1,y} \end{pmatrix} \right] \quad (24) \quad \forall m,y$$

$$\mathrm{ACTIVE\_INV}_{m,y} \leq \mathrm{tier\_lvl}_{m,num\_tiers,y} * \left(\mathrm{CUM\_TIER\_REACHED}_{m,num\_tiers,y} - \mathrm{CUM\_TIER\_REACHED}_{m,num\_tiers-1,y} \right) + \sum_{t=1}^{num\_tiers-1} \left[\left(\mathrm{tier\_lvl}_{m,t+1,y} - 1 \right) * \begin{pmatrix} \mathrm{CUM\_TIER\_REACHED}_{m,t,y} \\ -\mathrm{CUM\_TIER\_REACHED}_{m,t-1,y} \end{pmatrix} \right] \quad (25) \quad \forall m,y$$

$$\mathrm{CUM\_TIER\_REACHED}_{m,t+1,y} \geq \mathrm{CUM\_TIER\_REACHED}_{m,t,y} \quad (26) \quad \forall m,y,\ t<num\_tiers$$

$$\mathrm{MIN\_TIER}_{y} \leq \sum_{t=2}^{num\_tiers} \begin{pmatrix} \mathrm{t*CUM\_TIER\_REACHED}_{m,t,y} \\ -\mathrm{CUM\_TIER\_REACHED}_{m,t-1,y} \end{pmatrix} + \mathrm{CUM\_TIER\_REACHED}_{m,'1',y} \quad (27) \quad \forall m,y$$

4. New Constraints

We add the following constraints.

$$\mathrm{ACTIVE\_INV}_{m,y} = \sum_{t} (\mathrm{LAMBDA}_{m,t,y} * tier\_lvl_{m,t,y}) \quad (1) \quad \forall m,y$$

$$\mathrm{TIER\_ACHIEVED}_{m,y} = \sum_{t} (\mathrm{LAMBDA}_{m,t,y} * ord(t)) \quad (2) \quad \forall m,y$$

$$\mathrm{MIN\_TIER}_{y} \leq \mathrm{TIER\_ACHIEVED}_{m,y} \qquad (3)$$

$$\forall m,y$$

$$\sum_{t} \mathrm{LAMBDA}_{m,t,y} = 1 \qquad (4)$$

$$\forall m,y$$

$$\mathrm{LAMBDA}_{m,t,y} \leq \mathrm{TIER\_INDICATOR}_{m,t,y} + \mathrm{TIER\_INDICATOR}_{m,t+1,y} \qquad (5)$$

$$\forall m,t,y$$

$$t<num\_tiers$$

$$\sum_{t} \mathrm{LAMBDA}_{m,t,y} \leq \sum_{t} \mathrm{TIER\_INDICATOR}_{m,t,y} \qquad (6)$$

$$\forall m,y$$

$$t=num\_tiers$$

5. Discussion of New Constraints

(1) A constraint determines the LAMBDA multiplier used for munition m in year y and applied to the general capability level of the munition.

(2) A constraint determines the tier leveling terms of the LAMBDA multipliers for munition m and tier t.

(3) For each munition in each year, a constraint identifies the minimum capability tier achieved.

(4) For any given munition in a given year, the sum of the LAMBDA variables must equal 1. This constraint allows for fractional solutions.

(5) Sets an upper limit on LAMBDA for a particular munition m and tier t.

(6) Sets an upper limit on LAMBDA for the top tier. This constraint is required in addition to (5) to ensure the final tier is accounted for as there is only one break point in the top tier, that being the lower level.

III. IMPLEMENTATION AND APPLICATION

A. DATA AND SCENARIOS

The munitions data used in Section B of this Chapter was originally provide by Naval Operational Logistics Support Center (NOLSC), formerly Naval Ammunition Logistics Center (NALC), in Bruggeman [2003, Appendix A]. While the munitions data is based on real world data, the starting inventories have been altered by NOLSC from the original, classified, numbers. Procurement and maintenance budget parameters used are shown in Table 4 and are similar to those used by Bruggeman. While the maintenance figures are the same as used by Bruggemen, the procurement budget numbers are slightly increased in our application to account for anticipated increases in munitions procurement budgets in the coming years and as reflected in the President's FY05 budget. An updated discount rate of 2.53% is applied and is obtained from the Office of Management and Budget (OMB) website [OMB 2004].

		Year							
(all budget figures are in M$)		1	2	3	4	5	6	7	8
Procurement Budget - Upper Limit		1,250	1,250	1,250	1,250	1,250	1,250	1,250	1,250
Procurement Budget - Lower Limit		1,125	1,125	1,125	1,125	1,125	1,125	1,125	1,125
Maintenance Budget		30	40	50	60	70	80	90	100
Discount Rate	0.0253								

Table 4. Budget parameters for reformulated AIM and original models

Procurement budgets represent the portion of Navy Weapons Procurement budget accounted for by the nineteen munitions used in our work and derived from the President's FY05 budget. NOLSC determined the appropriate value for the nineteen munitions. Maintenance budget increases due to increasing numbers of munitions in the inventory as the procurement plan is executed. Discount rate is determined from the OMB Real Discount Rate.

B. IMPLEMENTATION

1. Speed

Multiple runs of AIM show the reformulation to be consistently faster (by a very wide margin) than its predecessor.

A typical run of the original model produces a procurement plan with an optimality gap (the relative difference between the actual capability of the solution and the best estimate of potential capability) of 50% in approximately forty minutes. The reformulated AIM generates a solution in less than two minutes with an optimality gap of 15%. To reach a 15% optimality gap with the old model typically requires a run of over two hours.

Using comparable optimality gaps, this increase represents a 98% reduction in computational time. Additionally, the increased speed allows for optimality bounds to be tightened even further yet still retain reasonable solve times. In the scenario used here, it is possible to produce a solution with an optimality gap of 10% in just under ten minutes.

2. Output Validity

We also want to ensure the output of the model is still valid and comparable to that of the original in terms of overall improvement in munitions capability. To do this we compare the capabilities of the nineteen munitions at the end of the eight-year planning period as generated using the original model versus that from reformulated AIM. Figures 5 and 6 illustrate this comparison and highlight the fact that, though different munitions are sometimes emphasized for improvement, overall the output of the two models is comparable. But, the overall measurement of munitions capability, the minimum tier level over all munitions, is higher with reformulated AIM.

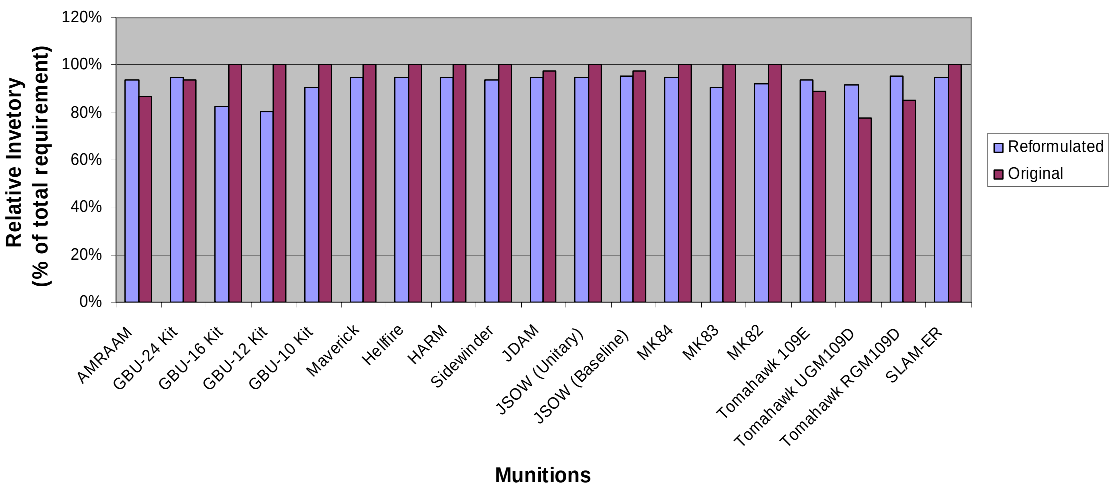

Figure 5. Comparison of original AIM and the reformulated model relative inventories at the end of Year 8

The relative inventory levels after eight years are shown for each weapon as calculated by original AIM and our reformulated version. While the original formulation shows measurably higher inventories (greater than 5%) for six of the munitions, the reformulated model is equal to or better in the remaining thirteen munitions. Also, the minimum is better in the reformulated model (GBU-12 at 81% vs. Tomahawk RGM109D at 78% in the original model).

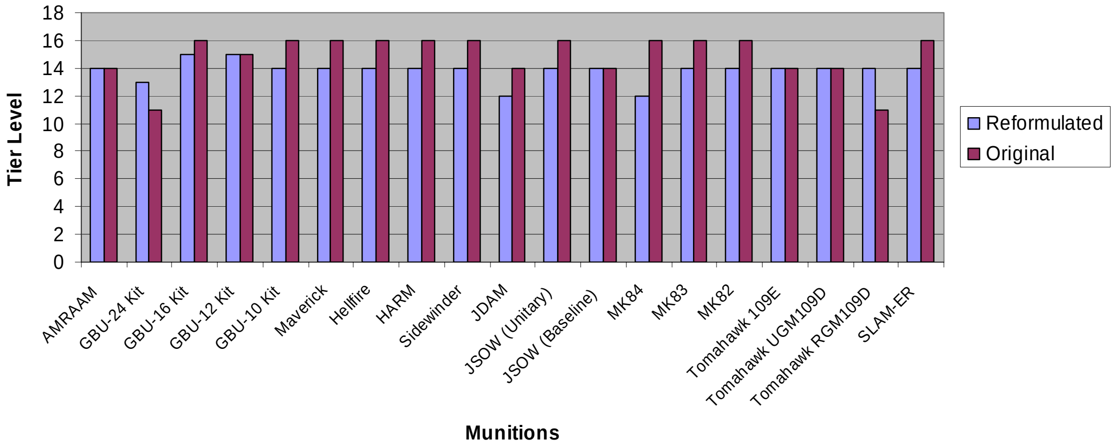

Figure 6. Comparison of original AIM and reformulated model tier levels after eight years

Comparing the tier levels achieved between the two models gives another indication of the validity of the reformulation. The original formulation generally does slightly better on an individual basis but overall the reformulated model has a higher minimum tier level (JDAM at 12 in the reformulated model vs. Tomahawk 109D at 11 in the original model). As our objective with this model is to maximize the minimum tier level, this indicates that we have done better in this regard (though original AIM may be able to achieve the same level but has not due to the integrality gap of its solution) in addition to greatly increased speed of computation.

3. Conclusion

The ability to bring down the upper bounds in our solution to the point of achieving a fifteen or ten percent gap in less than ten minutes is significant. Not only do we gain the direct benefit of reduced execution time, we can also be more confident in the quality of the solution.

C. APPLICATION OF REFORMULATED MODEL

The ability to quickly produce accurate solutions allows for the study of various budgeting and procurement scenarios in a very short period of time. Such flexibility provides the decision maker with a powerful tool with which to explore various alternatives with respect to procurement or maintenance funding, munitions mixes, or industrial base issues.

To highlight the capabilities of the new model we explore several procurement scenarios and their effect on munitions procurement plans.

We refer to the munitions data as previously given in the base case and then note excursions in each scenario. All parameters and planning factors are notional, unclassified and developed solely for these scenarios.

1. Critical Munitions Restock

In the first scenario we examine a case where a particular weapon, in our case the MK82, has a beginning inventory suddenly drop to zero. Figures 7 and 8 show the base case procurement plan for this munition given the default beginning inventory numbers for the MK82. In the base case, we observe a drop in capability in year three. This is due to large deliveries of previously purchased munitions in years one and two and practically none in year three. Alongside the base case is the modified procurement plan that results from a beginning inventory level of zero. One can clearly see the shift in procurements to the earlier years in order to rebuild inventory levels.

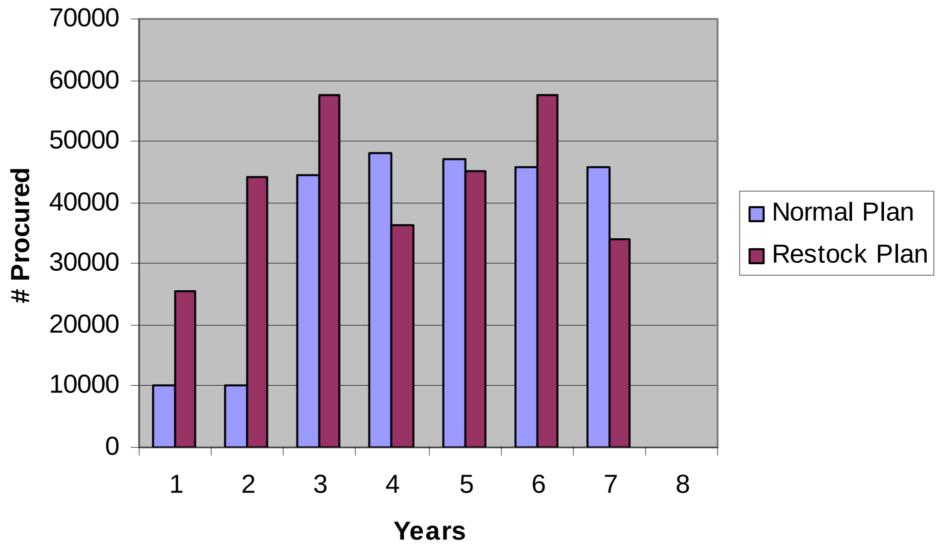

Figure 7. Comparison of normal initial inventory procurement plan versus single munition restock plan

The procurement policy for base case inventory levels is shown along with the required plan if a single munition inventory drops to zero at the beginning of the planning horizon. One sees the expected shift in procurement to earlier in the procurement plan in order to rebuild inventory levels.

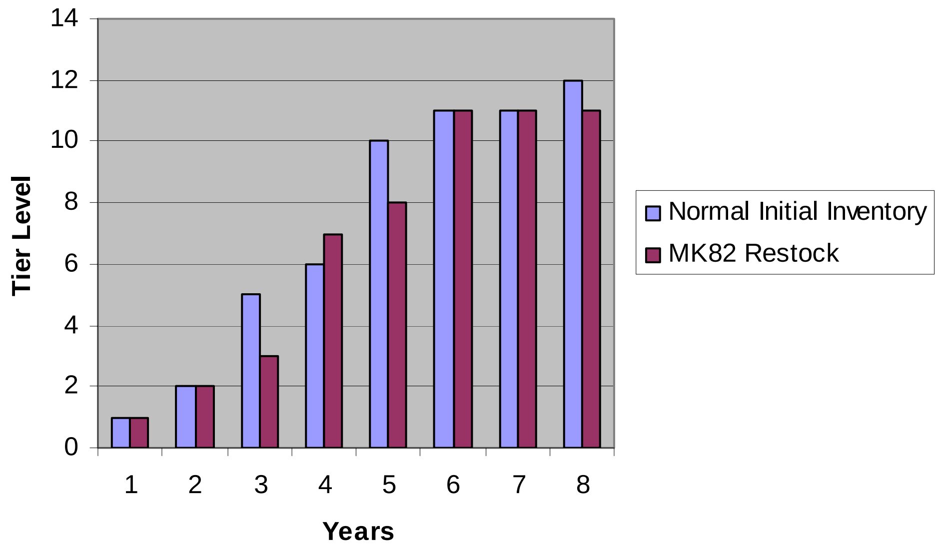

Figure 8. Comparison of overall munitions capability for base case inventory versus overall capability accounting for MK82 complete restock

The effect on the overall inventory due to the need to restock MK82 inventory from zero is visible here. Resources must be shifted to raise the MK82 capability which then drags the minimum overall capability down accordingly. The two plans achieve equality again in years six and seven.

2. Deferred Procurement Budget

Total funding for this scenario, $10B, remains the same for both funding plans. Funding for years one-through-four are reduced by $250M per year while years five-through-eight are increased by this same amount. One can easily see in Figure 9 the impact such a funding decision would have. While total capability is the same under either plan at the end of the eighth year, total inventory capability lags by two years with a very pronounced gap at the end of the fifth year. These results were achieved within 30 seconds.

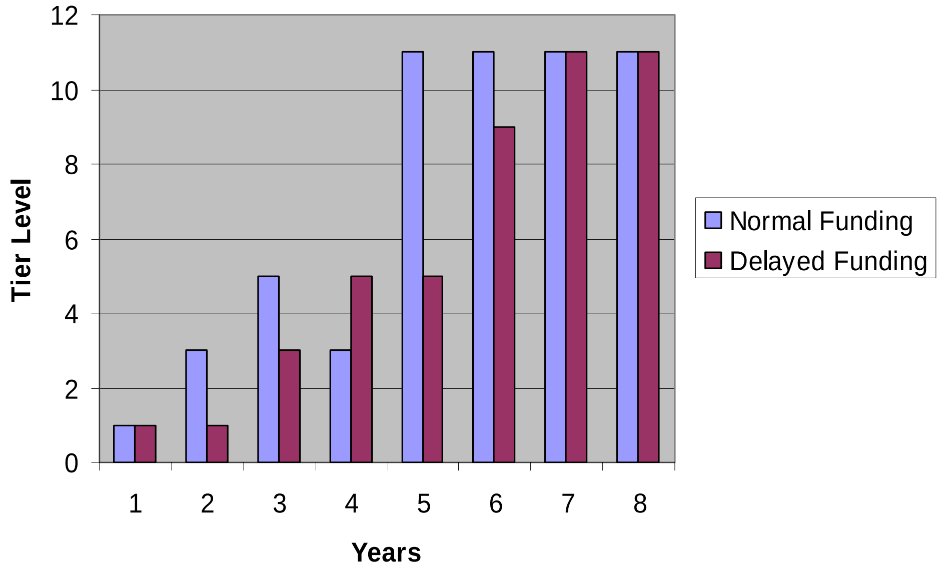

Figure 9. Comparison of base case funding plan versus delayed funding

With constant funding of $1.125B per year the minimum tier level is up to 11 by the end of year five, compared to tier 5 if some funding is delayed.

3. Front-Loaded Procurement Budget

Our third scenario considers a case where procurement funding is higher early in the planning years. For this scenario we decremented years four through six $250M each and distributed those funds equally among years one through three. We see the results of such a policy displayed in Figure 10 below along with the level funding plan. In something of a surprise we see no benefit in overall tier level capability for the early years despite the shift in funding. No real advantage is gained until year four but that is soon erased in year five and beyond.

This is a useful scenario in that the model has provided a non-intuitive insight: more funding early might not help as much as might be thought. It appears that the level of funding added in this scenario is insufficient to create a significant change in inventory capability. As with the earlier scenarios this one produced a solution in approximately thirty seconds.

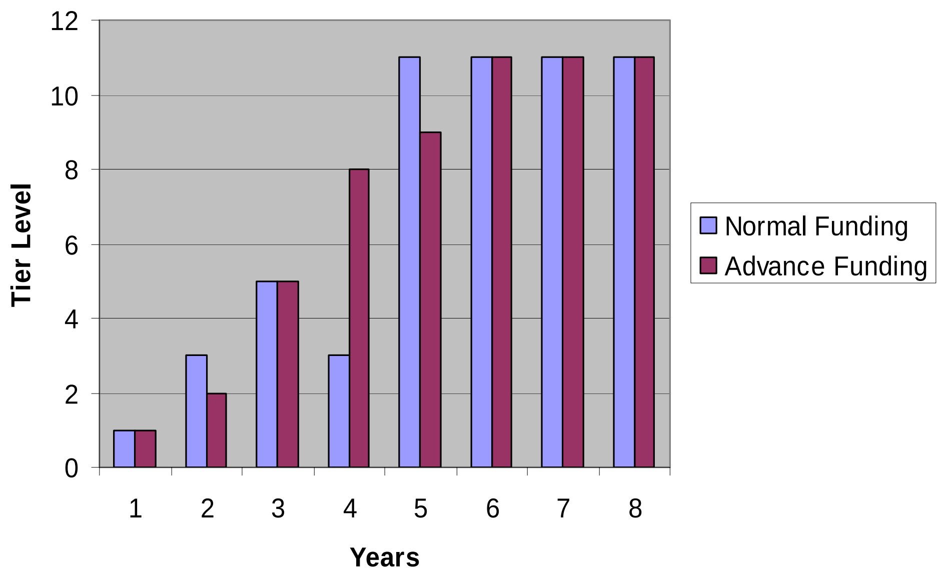

Figure 10. Comparison of base case plan versus advanced funding

The graph displays the effect of a base case plan versus a hypothetical plan that shifts some funding from years four through six to years one through three. It appears not enough funding was shifted in this scenario to show significant benefit in overall inventory capability.

4. Onetime Maintenance Increase

The final scenario we examine contains a one-time spike in maintenance funding early in the procurement plan. We want to see what affect, if any, such a change would have on the overall procurement plan.

We choose to increase maintenance funding in the first year from $30M to $100M. Figure 11 shows us that such a change has no dramatic effect other than to bring the total munitions capability up in a more gradual manner than with the base case funding plan. This is probably due to minor differences in the integer programming enumeration; in terms of the objective function, it is not a significantly better result.

This is a counter-intuitive result but one that might be explained by the relatively small number of munitions in the maintenance pipeline for the first three years of the procurement plan. Lacking sufficient munitions requiring maintenance on which to spend additional budget, no significant overall capability improvement is realized.

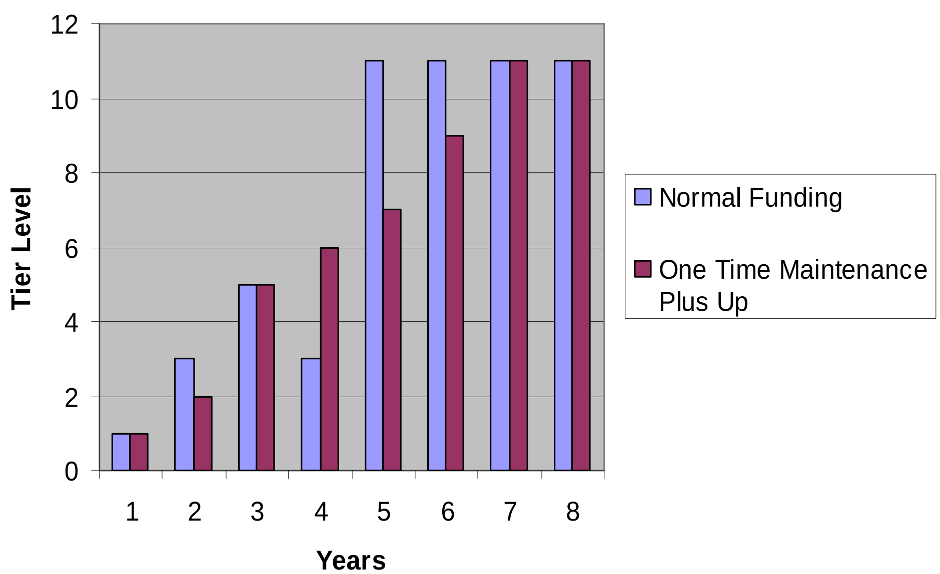

Figure 11. Comparison of base case plan versus level funding with a one-time maintenance increase

The one-time increase in maintenance funding occurs in year one of the plan and seems to have little impact at this level of increase. Overall performance is only slightly improved, and actually lags behind in the out-years.

IV. SUMMARY AND FUTURE RESEARCH

A. SUMMARY

Ordnance procurement planning is an extremely important problem for the U.S. Navy and the entire Department of Defense. It is a complex, multi-billion dollar problem that, until AIM, was not addressed optimally in a quantitatively measurable or financially constrained way.

AIM is a major step forward in addressing this problem, and we have significantly improved its computational speed.

B. RECOMMENDATION FOR FUTURE RESEARCH

Many aspects of AIM and the problem of ordnance procurement planning remain as productive areas for further research. While we concentrate on adding greater efficiency and detail to the model with respect to tier level formulation a similar focus could be applied to other components as well.

The industrial base portion of the model is a prime candidate for further research. AIM does a reasonable job of modeling this portion of the problem but far more detail exists in describing the manufacturing processes. In reality munitions are manufactured through a complex multi-component process that often involves several manufacturing sites around the country. Individual vendors have multiple munitions sharing facilities, and avoiding the “vanishing vendor” problem introduces other difficulties. These and other complexities in the industrial base have a direct impact on the economics of ordnance procurement planning and, therefore increased fidelity would certainly be of value.

Ordnance maintenance is another segment of the model that, due to its complexity, could benefit from further development. Maintenance costs and requirements are prime drivers in ordnance management and a greater understanding of exactly how that portion of the real world system functions could yield substantial improvement in how such characteristics are modeled in AIM.

Bruggeman developed a fast heuristic solver. We have chosen to forgo revising the heuristics because our integer linear program is now so much faster. A revised heuristic completely contained in, say, Microsoft Excel, would still be of value for the typical decision maker, and would make the application available to a far wider audience thus improving the chances of its eventual adoption by key decision makers.

APPENDIX. BRUGGEMAN AIM MODEL FORMULATION

The following formulation and write up are taken directly from [Bruggeman 2003].

A. INDICES AND SETS

$m \in M$ Munition, any munition for which NNOR requirements are generated, currently this is 190 possible munitions

$y \in Y$ Year of the planning horizon, $y=\{1,...,|Y|\sim 8\}$

$t \in T$ Tier level, $T=\{1,...,\text{num\_tiers}\}$

$l \in L$ Procurement pricing lot, $l=\{1,...,|L|\sim 10\}$, there may be up to ten different pricing lots identified for each munition

$f \in F$ Munition facility, $F=\{1,...,f_{max}\}$ where f_{max} is the total number of facilities being modeled

B. DATA

num_lots_m Number of procurement pricing lots actually used for munition m

$lot\_count_{m,l}$ Number of munition m in lot l

$lot\_cost_{m,l}$ Procurement cost for the full quantity of lot l of munition m

$unit\_cost_{m,l}$ Unit cost per munition m in lot l. Every munition must have at least two lots. For all m, $lot\_count_{m,'1'}=0$, and $lot\_cost_{m,'1'}$ is the penalty charged for violating the minimum sustaining rate for production. Subsequent lot counts and costs represent price reductions due to quantity purchasing. Counts and costs are cumulative; use these values as you would a table (interpolating linearly between given values) to determine the total cost for a desired quantity

$mun\_facility_{m,f}$	Value of 1 indicates munition m is produced at production facility f *and* maintained at maintenance facility f', 0 required otherwise
$min\_sust\_rate_m$	Minimum production Sustaining Rate (MSR) for munition m
$max\_prod\_rate_m$	Maximum Production Rate (MPR) for munition m
$prev\_procure_{m,y}$	Number of munition m to be delivered in year y from previous procurements (before beginning of AIM planning horizon)
$init\_invent_m$	Initial on-hand inventory of munition m at the beginning of the planning horizon
$delivery\_delay_m$	Number of years delay for delivery of new procurements of munition m

$init\_maint\_due_{m,y}$	Number of munition m in the initial inventory due for maintenance in year y
$maint\_cycle_{m}$	Time between scheduled maintenance for munition m, if no routine maintenance is required, this value must be large (>8)
$maint\_cost_{m}$	Unit cost of maintenance for munition m
$maint\_delay_{m}$	Number of years to return a maintained weapon m to the active inventory
$max\_maint\_rate_{m}$	Maximum annual maintenance rate for munition m
$min\_maint\_rate_{m}$	Minimum annual maintenance sustaining rate for munition m
$expend\_trng_{m,y}$	Expected annual training expenditures for munition m in year y
$expend\_ops_{m,y}$	Estimated annual operational expenditures for munition m in year y
$proc\_budget\_low_{y}$	Lower bound for annual procurement budget band in year y
$proc\_budget\_upp_{y}$	Upper bound for annual procurement budget band in year y
$maint\_budget_{y}$	Upper bound for annual maintenance budget in year y

disc_rate	8-year discount rate for future purchasing dollars from the OMB web site (http://www.whitehouse.gov/omb/circulars/a094/a94_appx-c.html), linearly interpolated between given values
mpr_lot_m	For munition m, lot number into which MPR falls
msr_lot_m	For munition m, lot number into which MSR falls
mpr_cost_m	Cost for the MPR quantity of munition m
msr_cost_m	Cost for the MSR quantity of munition m
$max\_prod\_cost_f$	Max annual production output of facility f, in total production costs
$min\_sust\_cost_f$	Min annual production output to sustain facility f, in total production costs
msr_pen_f	Monetary penalty for violation of facility f's MSR
$mpr\_pen\_rate_f$	Proportional additional penalty cost to facility f for exceeding its MPR
$max\_maint\_cost_f$	Max annual maintenance output of facility f, in total maintenance costs

min_maint_cost$_f$	Min annual output to sustain maintenance facility f, in total maintenance costs
excess_maint_rate	Proportional increase in maintenance costs for exceeding the maximum maintenance rate
persist	1 if this is to be solved as a persistent solution
cold2hot	1 to prohibit cold facilities from going hot in a designated number of years
hot2cold	1 to prohibit hot facilities from going cold in a designated number of years
cold2hot_time	Number of years to enforce cold to hot constraint
hot2cold_time	Number of years to enforce hot to cold constraint
change_limit	1 to enforce restrictions on changes in procurement quantities by year
change_percent$_y$	Limit, as a percentage, to the allowable change in procurements of each munition, from the incumbent solution, in year y
num_proc$_{m,y}$	Number of munition m procured in year y in the incumbent solution
of_wts$_y$	Objective function weights, by year y

$holding\_penalty_m$	Objective function penalty for "holding" a munition m in maintenance rather than performing the maintenance
$budget\_penalty_y$	Objective function penalty for underspending the procurement budget lower bound in year y
num_tiers	Number of tier levels in the tier formulation
$tier\_lvl_{m,t,y}$	Number of weapons of type m in year y required to reach tier t

C. VARIABLES

$PROCURED_{m,y}$	Number of munition m procured during year y
$LOT\_PROCURED_{m,l,y}$	Number of munition m procured from lot l in year y
$PROC\_COST_{m,y}$	Total cost of procurement of munition m in year y
$DELIVERED_{m,y}$	Number of munition m delivered during year y from both new procurement and maintenance
$ACTIVE\_INV_{m,y}$	Number of munition m in the usable inventory at the end of year y
$MAINT\_INV_{m,y}$	Number of munition m awaiting maintenance (not usable) at the end of year y

$MAINT\_DUE_{m,y}$	Number of munition m due for maintenance during year y
$MAINT\_RTN_{m,y}$	Number of munition m returned from maintenance (again usable) during year y
$MAINT\_SLACK_{f,y}$	Maintenance throughput of facility f below the minimum maintenance sustaining rate in year y, in total maintenance costs
$MAINT\_SURPLUS_{f,y}$	Maintenance throughput of facility f above the maximum maintenance rate in year y, in total maintenance costs
$MIN\_MAINT\_PEN_{f,y}$	Monetary penalty for violation of the minimum maintenance rate for facility f in year y
$MAX\_MAINT\_PEN_{f,y}$	Monetary penalty for violation of the maximum maintenance rate for facility f in year y
$OVERPROD_{f,y}$	Value of munitions procured in year y from facility f above the value of the Max Production Rate
$MPR\_PEN_{f,y}$	Amount of penalty paid for procurements in excess of MPR at facility f in year y
MIN_TIER_{y}	Minimum tier achieved of all munitions in year y

$SPEND\_SLACK_y$	Slack variable for spending below the procurement budget lower bound in year y
$PERS\_SLACK_{m,y}$	Slack variable for quantity of munition m by which persistence goals were not met in year y
$COLD\_SLACK_{f,y}$	Slack variable for persistence goals, a 1 indicates a failure to keep facility f "cold" in year y of the updated solution
$HOT\_SLACK_{f,y}$	Slack variable for persistence goals, a 1 indicates a failure to keep facility f "hot" in year y of the updated solution
$CUM\_TIER\_REACHED_{m,t,y}$	Binary variable, 1 if munition m is in tier t or below in year y
$LOT\_INDICATOR_{m,l,y}$	Binary variable, 1 if munition m is being procured in lot l during year y
$MEET\_MSR_{f,y}$	Binary variable, 1 if facility f satisfies its MSR in year y

D. CONSTRAINTS AND OBJECTIVE FUNCTION

MAXIMIZE

$$\sum_{y}\left[\text{of\_wts}_{y}*\left(\text{MIN\_TIER}_{y}-\sum_{f}\left(\text{COLD\_SLACK}_{f,y}+\text{HOT\_SLACK}_{f,y}\right)\right)\right]-\sum_{m,y}\text{PERS\_SLACK}_{m,y}$$

$$+\frac{\sum_{m,y}\left(\text{ACTIVE\_INV}_{m,y}-\text{budget\_penalty}_{y}*\text{SPEND\_SLACK}_{y}-\text{holding\_penalty}_{m}*\text{MAINT\_INV}_{m,y}\right)}{\sum_{m,y}\text{tier\_lvl}_{m,num\_tiers,y}}$$

subject to:

1\. $\text{ACTIVE\_INV}_{m,y} = \text{init\_invent}_{m}+\text{DELIVERED}_{m,y}$

2\. $-\text{MAINT\_DUE}_{m,y}-\text{expend\_trng}_{m,y}-\text{expend\_ops}_{m,y}$ $\quad \forall m,\ y=1 \quad (1)$

$$\text{ACTIVE\_INV}_{m,y} = \text{ACTIVE\_INV}_{m,y-1}+\text{DELIVERED}_{m,y}-\text{MAINT\_DUE}_{m,y}-\text{expend\_trng}_{m,y}-\text{expend\_ops}_{m,y} \qquad \forall m, y>1 \qquad (2)$$

$$\text{ACTIVE\_INV}_{m,y} \geq \text{tier\_lvl}_{m,'1',y} \qquad \forall m,y \qquad (3)$$

$$\text{DELIVERED}_{m,y} = \text{prev\_procure}_{m,y}+\text{PROCURED}_{m,y'}+\text{MAINT\_RTN}_{m,y''} \qquad \forall m,y \qquad (4)$$

$$\forall y'=y-\text{delivery\_delay}_{m}$$

$$\forall y''=y-\text{maint\_delay}_{m}$$

$$\text{MAINT\_INV}_{m,y} = \text{MAINT\_DUE}_{m,y}-\text{MAINT\_RTN}_{m,y} \qquad \forall m,y=1 \qquad (5)$$

$$\text{MAINT\_INV}_{m,y} = \text{MAINT\_INV}_{m,y-1}+\text{MAINT\_DUE}_{m,y}-\text{MAINT\_RTN}_{m,y} \qquad \forall m,y>1 \qquad (6)$$

$$\text{MAINT\_DUE}_{m,y} = \text{init\_maint\_due}_{m,y}+\text{DELIVERED}_{m,y'} \qquad \forall m,y \qquad (7)$$

$$\forall y'=y-\text{maint\_cycle}_{m}$$

$$\sum_{m}\left(\text{MAINT\_RTN}_{m,y}*\text{maint\_cost}_{m}*\text{mun\_facility}_{m,f}\right) \geq \text{min\_maint\_cost}_{f}-\text{MAINT\_SLACK}_{f,y} \qquad \forall f,y \qquad (8)$$

$$\sum_{m}\left(\text{MAINT\_RTN}_{m,y}*\text{maint\_cost}_{m}*\text{mun\_facility}_{m,f}\right) \leq \text{max\_maint\_cost}_{f}+\text{MAINT\_SURPLUS}_{f,y} \qquad \forall f,y \qquad (9)$$

$$MAX\_MAINT\_PEN_{f,y} = excess\_maint\_rate * MAINT\_SURPLUS_{f,y} \qquad \forall f,y \qquad (10)$$

$$MIN\_MAINT\_PEN_{f,y} = 1.05 * MAINT\_SLACK_{f,y} \qquad \forall f,y \qquad (11)$$

$$\sum_{m} \sum_{y'=1}^{y} \left(MAINT\_RTN_{m,y'} * maint\_cost_{m} * (1-disc\_rate)^{y'-1} \right) + \sum_{f} \sum_{y'=1}^{y} \left(\begin{array}{l} MAX\_MAINT\_PEN_{f,y'} + \\ MIN\_MAINT\_PEN_{f,y'} * (1-disc\_rate)^{y'-1} \end{array} \right) \le \sum_{y'=1}^{y} maint\_budget\_upp_{y'} \qquad \forall m,y \qquad (12)$$

$$PROCURED_{m,y} = \sum_{l=1}^{num\_lots_{m}} LOT\_PROCURED_{m,l,y} \qquad \forall m,l,y \qquad (13)$$

$$LOT\_PROCURED_{m,l,y} \ge \left(lot\_count_{m,l+1} - lot\_count_{m,l} \right) * LOT\_INDICATOR_{m,l+1,y} \qquad \forall m,y,\ l<num\_lots_{m} \qquad (14)$$

$$LOT\_PROCURED_{m,l,y} \le \left(lot\_count_{m,l+1} - lot\_count_{m,l} \right) * LOT\_INDICATOR_{m,l,y} \qquad \forall m,y,\ l<num\_lots_{m} \qquad (15)$$

$$LOT\_PROCURED_{m,l,y} \le \left(tier\_lvl_{m,num\_tiers,y} - lot\_count_{m,l} \right) * LOT\_INDICATOR_{m,l,y} \qquad \forall m,y,\ l=num\_lots_{m} \qquad (16)$$

$$\sum_{m} \left(PROC\_COST_{m,y} * mun\_facility_{m,f} \right) \le max\_prod\_cost_{f} + OVERPROD_{f,y} \qquad \forall f,y \qquad (17)$$

$$\sum_{m} \left(PROC\_COST_{m,y} * mun\_facility_{m,f} \right) \ge MEET\_MSR_{f,y} * min\_sust\_cost_{f} \qquad \forall f,y \qquad (18)$$

$$\sum_{m} \left(PROC\_COST_{m,y} * mun\_facility_{m,f} \right) \le MEET\_MSR_{f,y} * \sum_{m} \left(mun\_facility_{m,f} * tier\_lvl_{m,num\_tiers,y} \right) \qquad \forall f,y \qquad (19)$$

$$\mathrm{MPR\_PEN}_{f,y} = \mathrm{mpr\_pen\_rate}_f * \mathrm{OVERPROD}_{f,y} \qquad \forall f,y \qquad (20)$$

$$\mathrm{PROC\_COST}_{m,y} = \sum_{l=1}^{\mathrm{num\_lots}_m} \left(\mathrm{LOT\_PROCURED}_{m,l,y} * \mathrm{unit\_cost}_{m,l}\right) \qquad \forall m,y \qquad (21)$$

$$\sum_{m} \sum_{y'=1}^{y} \left(\mathrm{PROC\_COST}_{m,y'} * (1-\mathrm{disc\_rate})^{y'-1}\right) + \sum_{f} \sum_{y'=1}^{y} \left[\left(\mathrm{MPR\_PEN}_{f,y'} + \left(1-\mathrm{MEET\_MSR}_{f,y'}\right) * \mathrm{msr\_pen}_f\right) * (1-\mathrm{disc\_rate})^{y'-1}\right] \le \sum_{y'=1}^{y} \mathrm{proc\_budget\_upp}_{y'} \qquad \forall y \qquad (22)$$

$$\sum_{y'=1}^{y} \left[\left(\mathrm{SPEND\_SLACK}_{y'} + \sum_{m} \mathrm{PROC\_COST}_{m,y'}\right) * (1-\mathrm{disc\_rate})^{y'-1}\right] + \sum_{f} \sum_{y'=1}^{y} \left[\left(\mathrm{MPR\_PEN}_{f,y'} + \left(1-\mathrm{MEET\_MSR}_{f,y'}\right) * \mathrm{msr\_pen}_f\right) * (1-\mathrm{disc\_rate})^{y'-1}\right] \ge \sum_{y'=1}^{y} \mathrm{proc\_budget\_low}_{y'} \qquad \forall y \qquad (23)$$

$$\mathrm{ACTIVE\_INV}_{m,y} \ge \mathrm{tier\_lvl}_{m,'1',y} * \mathrm{CUM\_TIER\_REACHED}_{m,'1',y} + \sum_{t=2}^{num\_tiers} \left[\mathrm{tier\_lvl}_{m,t,y} * \left(\mathrm{CUM\_TIER\_REACHED}_{m,t,y} - \mathrm{CUM\_TIER\_REACHED}_{m,t-1,y}\right)\right] \qquad \forall m,y \qquad (24)$$

$$\mathrm{ACTIVE\_INV}_{m,y} \le \mathrm{tier\_lvl}_{m,num\_tiers,y} * \left(\mathrm{CUM\_TIER\_REACHED}_{m,num\_tiers,y} - \mathrm{CUM\_TIER\_REACHED}_{m,num\_tiers-1,y}\right) + \sum_{t=1}^{num\_tiers-1} \left[\left(\mathrm{tier\_lvl}_{m,t+1,y} - 1\right) * \left(\mathrm{CUM\_TIER\_REACHED}_{m,t,y} - \mathrm{CUM\_TIER\_REACHED}_{m,t-1,y}\right)\right] \qquad \forall m,y \qquad (25)$$

$$\mathrm{CUM\_TIER\_REACHED}_{m,t+1,y} \ge \mathrm{CUM\_TIER\_REACHED}_{m,t,y} \qquad \forall m,y,\ t<num\_tiers \qquad (26)$$

$$\text{MIN\_TIER}_y \le \sum_{t=2}^{num\_tiers} \begin{pmatrix} \text{t*CUM\_TIER\_REACHED}_{m,t,y} \\ \text{-CUM\_TIER\_REACHED}_{m,t-1,y} \end{pmatrix} + \text{CUM\_TIER\_REACHED}_{m,'1',y} \qquad \forall m,y \qquad (27)$$

Persistence constraints:
If persist=1 and change_limit=1 and change_percent$_y$>0,

$$\text{PROCURED}_{m,y}/\text{num\_proc}_{m,y} \ge 1\text{-change\_percent}_y\text{-PERS\_SLACK}_{m,y} \qquad \forall m,y \qquad (28)$$

If persist=1 and change_limit=1 and change_percent$_y$>0,

$$\text{PROCURED}_{m,y}/\text{num\_proc}_{m,y} \le 1\text{+change\_percent}_y\text{+PERS\_SLACK}_{m,y} \qquad \forall m,y \qquad (29)$$

If persist=1 and cold2hot=1 and cold2hot_time>y,

$$\text{MEET\_MSR}_{f,y} \le \sum_m \left(\text{num\_proc}_{m,y}\text{*mun\_facility}_{m,f}\right)\text{+COLD\_SLACK}_{f,y} \qquad \forall f,y \qquad (30)$$

If persist=1 and hot2cold=1 and hot2cold_time>y,

$$\text{MEET\_MSR}_{f,y} \ge \frac{\sum_m \left(\text{num\_proc}_{m,y}\text{*mun\_facility}_{m,f}\right)}{\sum_m \left(\text{num\_proc}_{m,y}\text{*mun\_facility}_{m,f}\right)+1}\text{-HOT\_SLACK}_{f,y} \qquad \forall f,y \qquad (31)$$

$\text{PROCURED}_{m,y}$, $\text{LOT\_PROCURED}_{m,l,y}$, $\text{PROC\_COST}_{m,y}$, $\text{DELIVERED}_{m,y}$, $\text{ACTIVE\_INV}_{m,y}$, $\text{MAINT\_INV}_{m,y}$, $\text{MAINT\_DUE}_{m,y}$, $\text{MAINT\_RTN}_{m,y}$, $\text{MAINT\_SLACK}_{f,y}$, $\text{MAINT\_SURPLUS}_{f,y}$, $\text{MIN\_MAINT\_PEN}_{f,y}$, $\text{MAX\_MAINT\_PEN}_{f,y}$, $\text{OVERPROD}_{f,y}$, $\text{MPR\_PEN}_{f,y}$, MIN_TIER_y, SPEND_SLACK_y, PERS_SLACK_y,
COLD_SLACK_y, $\text{HOT\_SLACK}_y \ge 0$ $\qquad \forall m,y,t,l \qquad$ (32)

$\text{CUM\_TIER\_REACHED}_{m,t,y}$, $\text{LOT\_INDICATOR}_{m,l,y}$,
$\text{MEET\_MSR}_{f,y}$ are Binary $\qquad \forall m,y,t,l \qquad$ (33)

E. DESCRIPTION

The objective function expresses the weighted sum of the annual minimum tier achieved, less penalties for violations of persistence, plus the sum of annual inventories as a proportion of the total desired inventory, less penalties for under spending on procurement and delaying maintenance.

Constraints:

(1-2) Together, these are inventory balance equations for each active (combat useable) munition.

(3) Each constraint requires that the minimum active inventory of a munition be maintained every year.

(4) Each constraint determines the number of a newly produced or maintained munition that is delivered in a given year.

(5-6) Together, these are inventory balance equations for a unusable munition that is waiting for maintenance.

(7) Maintenance scheduling equations; these determine the number of a munition that are due for maintenance in a given year.

(8-9) These elastic constraints enforce the maintenance base for the minimum and maximum maintenance throughput, in cost, in a given year for a given facility. A violation ($MAINT\_SLACK_{f,y}$ and $MAINT\_SURPLUS_{f,y}$) results in an increased maintenance cost.

(10-11) These equations determine the penalties for a violation of a maintenance base constraint.

(12) Each constraint limits cumulative maintenance spending (including penalties) by the cumulative maintenance budget.

(13) This equation determines the total number of a munition procured in a given year by summing procurements over all individual lots.

(14-16) Together, these constraints require that an individual lot procurement is no larger in count than the count of the entire lot (or the NNOR total requirement when purchasing from the last lot) and that a munition may not be procured from the next lot without procuring the entire previous lot.

(17) Each elastic constraint restricts procurement production at a facility by the maximum production rate (MPR). A violation ($OVERPROD_{f,y}$) results in a penalty which increases procurement cost.

(18-19) Together, these constraints determine whether the minimum sustaining production rate (MSR) for a facility has been met. A failure to meet the MSR results in a penalty on overall procurement spending.

(20) Each equation determines the penalty for a violation of a facility's MPR.

(21) Each equation determines the total cost of new procurement of a single munition in a given year.

(22-23) Together these constraints enforce the upper and lower bounds on cumulative procurement budget spending, discounted for future years and including penalties.

(24-25) Together, these constraints determine which tier has been reached based on a current (active) inventory count.

(26) These constraints require the tier reached indicator variable to be non-decreasing.

(27) Each constraint determines the minimum tier achieved in a given year.

(28-29) These constraints are active only when a persistent recommendation is desired. Together they require the quantity of a munition procured in a given year to be within a relative range of the quantity from the original recommendation.

(30-31) These elastic constraints require that a facility does not change status in the revised plan from "cold" to "hot" or "hot" to "cold" for a designated number of years. A violation ($COLD\_SLACK_{f,y}$ and $HOT\text{-}SLACK_{f,y}$) is penalized in the objective function.

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